University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2014**

Problem Set #2

(due 11/25/14)

1. Consider an economy in which relative producer prices are fixed and there are H identical households, each with the following utility function in household consumption of goods 1 and 2, c_1 and c_2, household leisure, l, and aggregate consumption of good 1, $C_1 = Hc_1$:

$$U(c_1, c_2, l, C_1) = c_1^{\alpha_1} c_2^{\alpha_2} l^{1-\alpha_1-\alpha_2} C_1^{-\beta}$$

Each household maximizes this utility function subject to the budget constraint:

$$p_1 c_1 + p_2 c_2 + wl = y$$

where y equals the value of the household's labor endowment, $w\bar{L}$, less any lump-sum taxes paid to the government. In its optimization process, the household ignores the effect of its own consumption of good 1 on C_1, i.e., it treats C_1 as fixed when choosing c_1.

A. Solve for the household's indirect utility function, conditional on the value of C_1, $V(p_1, p_2, w, y; C_1)$. Use the household's demand function for c_1 and the fact that all households are identical to express C_1 in terms of income and prices, and substitute this expression for C_1 into the indirect utility function to obtain an expression for individual utility that is solely dependent on prices and income, $\tilde{V}(p_1, p_2, w, y)$. Letting the social welfare function be the sum of the utilities of the H identical households, use your expression for $\tilde{V}(\cdot)$ to obtain an expression for social welfare in terms of prices and aggregate income $Y = Hy$, i.e., $W(p_1, p_2, w, Y)$.

B. Let labor be the numeraire (w =1) and let *producer* prices for goods 1 and 2 be q_1 and q_2. Suppose that the government raises revenue R for public expenditures (which don't affect utility directly) with uniform lump-sum taxes and taxes on goods 1 and 2. Let θ_i be the proportional tax on good i, i.e., $\theta_i = (p_i - q_i)/p_i$ or $p_i = q_i/(1-\theta_i)$. Solve for the optimal values of θ_1 and θ_2, showing that the tax on good 2 is zero and that the tax on good 1 is β/α_1. (*Hint*: use the definition of Y to express it in terms of R, θ_1 and θ_2, insert the result into your expression for $W(\cdot)$ and maximize welfare directly with respect to the taxes.)

C. Now, suppose government must raise R without lump-sum taxes. Show that the ratio of consumer prices should be the same as in part B, i.e., that $\frac{q_1/(1-\theta_1^*)}{q_2/(1-\theta_2^*)} = \frac{q_1/(1-\theta_1^p)}{q_2}$, where $\theta_1^p = \beta/\alpha_1$ is the Pigouvian tax. (*Hint*: one can show this by comparing first-order conditions; it is not necessary to solve completely for taxes.)

2. In class, we showed that a consumption tax is equivalent to a tax on labor income plus a tax on existing assets. That derivation assumed that assets took the form of homogeneous capital. This question reconsiders the issue for a wider class of assets.

A. Write down the budget constraint, expressing consumption in terms of labor income and assets, for a household that lives for two periods, supplies labor L in the first period for wage w, has initial real assets in the first period, A, and consumes goods in both periods, c_1 and c_2, with consumption and assets having price p. Assume the household faces a tax at rate t on capital income and labor income and that saving yields a before-tax return r.

B. Only relative prices matter in part A, i.e., the price *level* is indeterminate. Suppose now that the household initially holds two types of assets, real capital and nominal bonds, each yielding r. Rewrite the budget constraint for this case, with A the fixed *real* quantity of capital and B the fixed *nominal* quantity of bonds. Assume the price level is the price of consumption goods, p, in both periods, which implies that there is no inflation.

C. Suppose that, at the beginning of period 1, the government replaces the income tax with a sales tax at rate τ on consumption in both periods, and that the real wage (in terms of the producer price of consumption) and the real before-tax return, r, remain the same. Also assume that the price level *net* of sales tax does not change in either period, i.e., remains equal to p. Rewrite the budget constraint from part B for this tax system, showing that the consumption tax is equivalent to a tax on labor income plus all existing wealth.

D. Now, change the assumption about the price level in part C. Suppose that, when the sales tax is imposed, the Fed uses monetary policy to keep the consumer price index (which now includes the sales tax) at its original value. How does your answer to part C change?

E. Now, go back to the price level assumption from part C, but assume that bond interest is exempt from the initial income tax (as is true for bonds issued by U.S. state and local governments), so that in equilibrium under the income tax bonds yield a <u>before-tax</u> return of $r(1-t)$. Suppose also that bonds are consols, i.e., of infinite duration with constant nominal payments over time. How does your answer to part C change?

3. Suppose that a risk-neutral investor seeking to maximize terminal wealth faces a tax rate of c on capital gains, while facing a tax rate of $t \geq c$ (i.e., getting a refund at rate t) on long-term capital losses. The investor has an asset originally purchased for P_0 that is now worth $P_1 > P_0$, and must decide whether to (1) sell the asset now, pay a tax on $(P_1 - P_0)$ at rate c, and reinvest the remaining proceeds for one more period, or (2) continue holding the asset for one more period. In either case, the rate of return over the next period is r, which is stochastic. Under choice (1), subsequent gains will be taxed at c and subsequent losses will be taxed at t. Under choice (2), total gains, $(P_1(1+r) - P_0)$ will be taxed at rate c, for we assume that $P_1(1+r) > P_0$ even if r is negative.

   A. Derive a necessary and sufficient condition for the investor to realize the gain now, expressed as an inequality in terms of some critical value, say R^*, of the ratio $R = P_1/P_0$.

   B. Show that $dR^*/dc < 0$, starting from the case in which t and c are initially equal.

   C. Also starting from the case in which $t = c$, show that $dR^*/dt > 0$. Explain your result.